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Engineering Notes

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Time-Critical Low-Thrust Orbit Transfer Optimization

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I. Introduction

FOR near-Earth orbit transfers by the electric orbital transfer vehicle (OTV), for which thruster systems are available with specific impulse between 500 and 7000 s,[1,2] the electric powerplant/propulsion system is designed to achieve best use of the spacecraft. Various approaches have been considered,[3,4] including variable thrust programming to minimize propellant consumption.[5–7] In this Note, we consider only constant thrust transfers between orbits.

The most elementary model for an electrically propelled spacecraft represents the initial spacecraft mass m_o as a sum of three masses: propellant mass m_p, the propulsion system mass m_{pow}, and the transferred payload mass m_t.

$$m_o = m_p + m_{\text{pow}} + m_t \tag{1}$$

We introduce expressions for the propellant mass and powerplant mass in terms of the characteristic velocity Δv, exhaust velocity u_e, thrust T, efficiency η, and powerplant specific mass α (kg/W), where α includes powerplant, thruster, structure, and tanks:

$$m_p/m_o = 1 - e^{-\Delta v/u_e} \tag{2}$$

$$m_{\text{pow}}/m_o = \alpha T u_e / 2\eta m_o \tag{3}$$

which are combined with Eq. (1) to give an expression for transferred mass:

$$m_t/m_o = e^{-\Delta v/u_e} - \frac{\alpha T u_e}{2\eta m_o} \tag{4}$$

The powerplant mass fraction is a function of the exhaust velocity u_e and the thruster efficiency $\eta(u_e)$. The power P and thrust are assumed constant, resulting in constant u_e and fixed η. Since the electric thruster type (arcjet, MPD, ion, etc.) can be selected to maximize η at optimum u_e, it is assumed for simplicity that η is a constant, independent of u_e.

We consider a simple electric LEO-GEO-LEO round-trip transfer, for which the mass fraction m_t/m_o is transferred to geosynchronous Earth orbit (GEO) and dropped off, and only the powerplant mass is returned to low Earth orbit (LEO). The mass transferred to GEO by the round-trip transfer vehicle is

$$m_t/m_o = e^{-\Delta v/u_e} - \left(\frac{\alpha T u_e}{2\eta m_o}\right) e^{\Delta v/u_e} \tag{5}$$

In Eq. (5), Δv is the one-way characteristic velocity, with Δv and u_e assumed equal on the LEO-GEO and GEO-LEO legs. Optimum powerplant mass fraction [Eq. (3)] is determined by equating the u_e derivative of Eq. (5) to zero, giving for the round trip

$$(m_{\text{pow}}/m_o)^{\text{opt}} = \left(\frac{\Delta v/u_e}{1 - \Delta v/u_e}\right) e^{-2\Delta v/u_e} \tag{6}$$

The mass transferred for the round-trip then becomes

$$(m_t/m_o)^{\text{opt}} = \left(\frac{1 - 2\Delta v/u_e}{1 - \Delta v/u_e}\right) e^{-\Delta v/u_e} \tag{7}$$

From Eq. (7), high mass transfer rates require relatively high specific impulse. For an electric LEO-GEO mission with a 29-deg plane change from a 500-km orbit, we take a conservative value for Δv of 6000 m/s,[4] slightly above the 5850 m/s predicted by Edelbaum.[8] For $m_t/m_o = 0.50$, Eq. (7) predicts $I_{\text{sp}} = 2354$ s for the round trip, with $m_{\text{pow}}/m_o = 0.209$ and $m_p/m_o = 0.291$.

This level of I_{sp} implies the acceptance of a relatively long trip time, which may be difficult to justify economically, even for the high fraction of mass transferred. Trip time is calculated assuming constant power, η and u_e, with the assumption of continuous thrust T, and is related to the propellant mass by an expression of the form[4]:

$$t_T = \frac{m_p}{dm/dt} = \frac{u_e m_o}{T}(1 - e^{-\Delta v/u_e}) \tag{8}$$

Trip time for a round trip is the sum of two one-way transfers, where the return trip time is expressed in terms of the return launch mass m_r from GEO:

$$m_r/m_o = e^{-\Delta v/u_e} - m_t/m_o \tag{9}$$

The round-trip time is then

$$(t_T)^{rt} = \frac{\alpha u_e^3 (1 - \Delta v/u_e)}{2\eta \Delta v}(e^{\Delta v/u_e})(e^{\Delta v/u_e} - 1) \times (1 + e^{-\Delta v/u_e} - m_t/m_o) \tag{10}$$

where $\Delta v/u_e$ is determined from Eq. (7). For $m_t/m_o = 0.50$ with $\Delta v = 6000$ m/s, $\alpha = 40$ kg/kW and $\eta = 0.50$; $I_{\text{sp}} = 2354$ s and $m_r/m_o = 0.271$. The round-trip time is then 343 days.

A trip time of this length is unrealistic in many situations. For a commercial electric OTV capable of a large number of orbit transfers, the financial costs of interest on debt, ground control personnel, and lost satellite revenue dictate lower transfer times. To minimize the cost of operating the OTV, it is required to have both high specific impulse to maximize payload and high thrust to reduce trip time. These simultaneous requirements conflict in a power-limited system.

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II. Transfer Rate Optimization

Trip time is taken into account by maximizing the mass transfer rate TR, defined as transfer mass fraction divided by the trip time[9]:

$$TR = (m_t/m_o)/t_T \tag{11}$$

Thrust, power, and efficiency are assumed constant, resulting in constant exhaust velocity u_e. For a one-way LEO-GEO transfer,

$$\frac{\alpha(\Delta v)^2 TR}{2\eta} = \left(\frac{m_{\text{pow}}}{m_o}\right)\left(\frac{\Delta v}{u_e}\right)^2 \frac{(e^{-\Delta v/u_e} - m_{\text{pow}}/m_o)}{1 - e^{-\Delta v/u_e}} \tag{12}$$

The round-trip transfer rate, with $(t_T)^{rt}$ expressed by Eq. (10), is

$$\frac{\alpha(\Delta v)^2 TR}{2\eta} = \left(\frac{m_{\text{pow}}}{m_o}\right)\left(\frac{\Delta v}{u_e}\right)^2 \times \frac{\left[e^{-\Delta v/u_e} - (m_{\text{pow}}/m_o)e^{\Delta v/u_e}\right]}{\left[1 - e^{-\Delta v/u_e} + (m_{\text{pow}}/m_o)(e^{\Delta v/u_e} - 1)\right]} \tag{13}$$

Equation (13) for TR is plotted in terms of the variables m_{pow}/m_o and $\Delta v/u_e$ in Fig. 1. It is clear from Fig. 1 that a maximum in TR exists. Equating the (m_{pow}/m_o) partial derivative of Eq. (13) to zero gives an expression for m_{pow}/m_o:

$$\left(\frac{m_{\text{pow}}}{m_o}\right)^{\text{opt}} = \left[(1 + e^{-\Delta v/u_e})^{1/2} - 1\right] e^{-\Delta v/u_e} \tag{14}$$

Substituting into Eq. (13) and simplifying gives the optimum round-trip transfer rate:

$$\frac{\alpha(\Delta v)^2 TR}{2\eta} = \left(\frac{\Delta v}{u_e}\right)^2 \frac{\left[2 + e^{-\Delta v/u_e} - 2(1 + e^{-\Delta v/u_e})^{1/2}\right]}{(e^{\Delta v/u_e} - 1)} \tag{15}$$

The maximum in TR from Eq. (15) occurs at $(\Delta v/u_e)^{\text{opt}} = 0.4319$. The maximum round-trip transfer rate is $\alpha(\Delta v)^2 TR/2\eta = 0.02790$. Optimum round-trip transfer and powerplant mass fractions are $m_t/m_o = 0.3650$; and $m_{\text{pow}}/m_o = 0.1845$. The propellant mass fraction is $m_p/m_o = 0.4505$.

The optimum return mass fraction from Eq. (9) is $m_r/m_o = 0.284$. For the previously assumed α, Δv, and η, the optimum I_{sp} is 1417 s, giving a LEO-GEO trip time of 170 days, a return GEO-LEO trip time of 48 days, and a total time of 218 days. This time is proportional to α/η and can be reduced with lighter, more efficient powerplant technology than that assumed here.

III. Comparison with Chemical OTV

The performance of the transfer rate-optimized electric OTV can be compared to a chemical OTV on the basis of Earth surface launch mass. For a reusable OTV the dry mass is amortized over a large number of flights, becoming a small fraction per flight. Over many flights the OTV is resupplied only with propellant and transfer mass, and we can define a launch penalty as

$$\text{launch penalty} = \frac{\text{kg launched}}{\text{kg transferred}} = 1 + \frac{\text{propellant mass}}{\text{transfer mass}}$$

For the round-trip transfer rate-optimized OTV discussed above with $m_p/m_o = 0.4505$ and $m_t/m_o = 0.3650$, the electric launch penalty is $(1 + m_p/m_t)_{\text{elec}} = 2.234$ (kg launched/kg transferred).

The chemical launch penalty is calculated for an advanced Centaur-based OTV, for which the dry mass fraction of engines, structure, and tanks is assumed as $m_{\text{dry}}/m_o = 0.06$.[10] For a LEO-GEO transfer with $\Delta v = 4200$ m/s (Ref. 11) and an H_2-O_2 specific impulse of 465 s, the round-trip chemical mass fractions are transfer mass, $m_t/m_o = 0.247$; propellant mass, $m_p/m_o = 0.693$; and dry mass, $m_t/m_o = 0.060$. The chemical launch penalty is $(1 + m_p/m_t)_{\text{chem}} = 3.804$. Comparing the electric launch case to the chemical case:

$$\frac{\text{chemical launch penalty}}{\text{electric launch penalty}} = 1.70$$

For this example, the reusable chemical OTV requires 70% more launch mass or a 70% larger launch vehicle to transfer the same mass from LEO to GEO as the electric system.

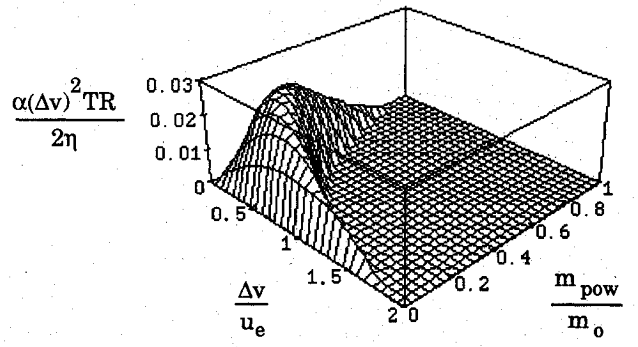

Fig. 1 Round-trip transfer rate for constant thrust as a function of powerplant mass ratio m_{pow}/m_o and $\Delta v/u_e$. Maximum $\alpha(\Delta v)^2 TR/2\eta = 0.02790$ occurs at $m_{\text{pow}}/m_o = 0.184$ and $\Delta v/u_e = 0.432$.

Table 1 Comparison of round-trip electric and chemical OTV mass fractions for $\Delta v = 6000$ m/s, $\alpha = 40$ kg/kW, and $\eta = 0.50$

Transfer type	Maximum mass	Maximum mass rate	H_2-O_2 chemical
Powerplant, m_{pow}/m_o	0.184	0.184	N/A
Propellant, m_p/m_o	0.247	0.450	0.693
Transfer mass, m_t/m_o	0.548	0.365	0.247
Specific impulse, s	2732	1417	465
Launch penalty, $1 + m_p/m_t$	1.43	2.23	3.80
Trip time, days	444	218	<1
Round-trip Δv^{rt}, m/s	7600	8300	5380

IV. Discussion of Results and Conclusions

We have employed two different approaches to optimize the mass distribution of a simple electric OTV for orbital mass transfer, first optimizing the mass transfer fraction and second optimizing the mass transfer per unit time. It is instructive to compare these two results for round-trip missions assuming the same powerplant mass fraction, as shown in Table 1.

Table 1 shows the tradeoff between the two electric optimization schemes, and the comparison to a chemical system. The round-trip Δv^{rt} is based on I_{sp} and the propellant fraction m_p/m_o at LEO. Optimizing transfer mass is seen to reduce the launch penalty by a third compared to mass rate, but doubles the trip time.

Depending on the relative costs associated with the launch vehicle and with an extended-time low thrust transfer, the lowest total cost for a particular OTV will fall between the maximum mass and maximum mass rate cases. Since the trip time cost increases linearly with the powerplant specific mass α, an intelligent choice of specific impulse and thruster type cannot be made until these relative costs and α are known.

For the round-trip LEO-GEO transfer with $\Delta v = 6000$ m/s, the highest transfer rate is achieved with an I_{sp} of 1417 s. This I_{sp} is above the efficient range of operation for arcjets and below that for ion engines, suggesting the need for a new type of electric thruster in the intermediate specific impulse range.

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Axial Compression Corner Flow with Shock Impingement

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Introduction

SEVERAL corner flow studies have been conducted by many researchers, both theoretically and experimentally, from simple, sharp leading-edge flat plates intersecting at right angles to more complex compression corners formed by unsymmetrical wedges intersecting at various angles.[1] Most of the studies focused on characterizing the flow structure with little, if any, effort on determining aerodynamic heating. Therefore, very little heat transfer data is available on axial corner flow. For corner flows with shock impingement, a situation that would exist in the engine inlet of a hypersonic vehicle, no experimental heat transfer data have been reported in the literature. This Note presents a portion of such experimental results, which are the first of their kind, from a 90-deg axial compression corner with external oblique shock impingement. A more detailed presentation of this study is given in Ref. 2.

Experimental Program

A sketch of the model with pertinent dimensions and coordinate system is shown in Fig. 1. The model consists of a forebody, cowl, and strut. The forebody has a 10-deg wedge section for the forward half of its total length of 18 in. The cowl has a 10-deg wedge angle and a length of 9 in. The strut has a 5-deg half-angle with 4.4 in. and 2.8 in. of height at the leading and trailing edges, respectively. The model was tested in the NASA Langley 20-in. Mach 6 wind tunnel[3] at a stagnation temperature of 420°F and a freestream unit Reynolds number of 3.35×10^6/ft with and without boundary-layer trips.

The surface heat transfer distribution was obtained using the phase-change paint technique described by Jones and Hunt.[4] The painted model was injected into the test stream and subjected to aerodynamic heating until the paint melted. A 35-mm color camera recorded the progression of the paint melt line at a rate of 30 frames/s. The local heat transfer coefficient h at the melt line was calculated from the analytical solution to the transient, one-dimensional heat conduction equation for a semi-infinite slab given in Ref. 3. The error in the calculated h using the phase-change paint method can be as large as 30%.[4,5] However, even with this degree of error, this technique is still quite useful to obtain the heating pattern, determine salient heating features such as localized "hot spots," and provide at least a correct order of magnitude estimate of the quantitative value of the heat transfer.

Surface oil-flow patterns were obtained using a mixture of titanium dioxide and silicon oil distributed over the model surface in a random dot pattern. Oil-flow development in the compression corner was captured using a 35-mm camera operating at 30 frames/s.

Results and Discussion

The heating rate contours computed from the phase-change paint experiments for the sharp-strut model without boundary-layer trips are shown in Fig. 2. The normalizing value of h_{ref} corresponds to the undisturbed laminar heat level at a distance of 4.4 in. along the 10-deg cowl from its leading edge, which is also the height of the cowl at the leading edge. Two distinct peak heating areas are shown on the cowl surface near the corner and downstream of the shock impingement region. The

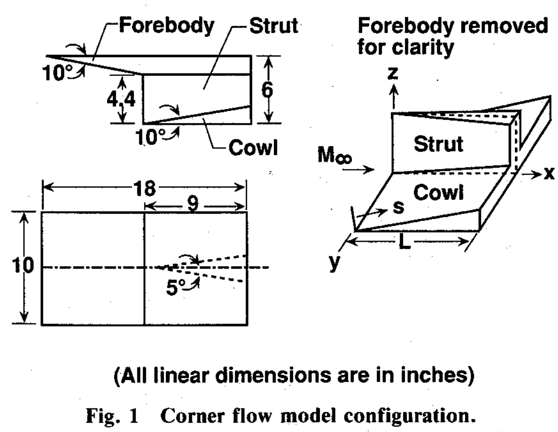

Fig. 1 Corner flow model configuration.

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